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# Periodic driving controls random motion of Brownian steppers

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**Abstract**

We study the motion of Brownian steppers, which are objects moving unidirectionally by discrete steps. A single step is composed of two processes. An activation process describing the random attachment of a fuel molecule is followed by a conformational change of the stepper, leading to the forward motion. Whereas activation is given by a Markovian rate process, the forward motion is defined by a gamma distribution. In this paper we propose a periodic modulation of the fuel concentration in order to control the random motion of the stepper. We show that the driving may reduce the fluctuations of the stepper. Corresponding minima of the diffusion coefficient and maxima of the Péclet number prove the regularity of the motion.

## 1. Introduction

For many physicists the term 'molecular motor' is associated with a Brownian ratchet. The latter is a simple model consisting of a particle in a spatially asymmetric time-dependent potential, subject to friction and thermal noise. This model became popular after Magnasco's 1993 work [1] and an abundant literature is devoted to the questions of rectified current (particle's velocity), thermodynamic efficiency, current reversals and many other intriguing properties of such systems both in underdamped and in overdamped regimes, under adiabatic or non-adiabatic modulation conditions, see [2–4] for reviews, and references therein.

However, the majority of real molecular motors powering our cells and their organelles are not rectifying fluctuations. They are more similar to deterministically working car motors, whose functioning implies a sequence of well defined processes, which, in an analogy with a heat engine, are called strokes; see e.g. [5–7]. These motors hardly show any reversal of their motion and are highly efficient and best suited for performing their well defined simple task.

There is, however, a considerable difference between the method of functioning of molecular motors [8] and macroscopic ones: due to their microscopic size, the importance of inertia and masses (being proportional to $L^3$ with $L$ being the size of the system) is negligible compared to the importance of friction, which, in the Stokes case, is proportional to $L$ itself. Another difference is that the motor is so tiny that the influence of the thermal agitation of

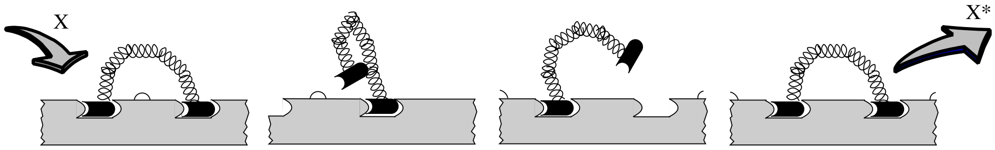


**Figure 1.** One step of the Brownian stepper. The step is induced by the binding of a fuel molecule $X$ according to a rate process with a rate $\gamma(t)$ which is proportional to the concentration $X(t)$ of the fuel molecules. Afterwards the motor molecule undergoes conformational changes, thereby releasing the used fuel $X^*$ and advancing by one step length $\mathcal{L}$. These conformational changes take some time $\tau$ which is distributed according to $w(\tau)$.

the molecules of the surrounding medium cannot be neglected. Thus, although the strokes themselves are well defined, we cannot neglect the effects of randomness introduced by their impacts. These are constructively used by the motors, whose working cycle might include thermally activated or diffusive steps; here we note that at the scales considered diffusion (with the typical displacement going as a square root of $t$) might be a fast process compared to deterministic sliding, with displacement going as $t$. Bier coined a description of such a motor (a simplified model of a two-headed kinesin walking along a biopolymer microtubule) as a Brownian stepper [9].

The paradigmatic model of a Brownian stepper might be used as a prototype of tiny engines on the nano-scale. Thus the question of the possibility of controlling the motion of a stepper can be posed. Such control cannot be easily realized through changing the properties of the molecule itself. It will be much easier to modulate the properties of the surrounding medium by changing, for example, the concentrations of some molecules (fuel molecules or special transmitters), just as adopted in biological prototypes.

This is exactly the mechanism of control we consider in some detail in the present work. We namely consider the influence of a periodic modulation of the fuel molecule concentration on the transport properties of the stepper. The transport properties of interest are the mean velocity $v$ and the effective diffusion coefficient $D_{\text{eff}}$ of the molecular motor. The first one determines the effectiveness of transport, and the second one, describing the spread of the actual positions in different realizations around the mean, gives us the measure of how accurately this molecular step motor works. The characteristic measure of this precision is the dimensionless Péclet number, $Pe = \mathcal{L}v/D_{\text{eff}}$, where $\mathcal{L}$ is the length of one step [10].

## 2. The model

A Brownian stepper is a molecular motor which moves along a track in discrete forward steps (figure 1). Each step is induced by the consumption of a fuel molecule. This initiation of a step happens according to a rate process with a rate $\gamma$, which is proportional to the concentration of the fuel molecules. After a step is triggered, the motor molecule performs some conformational changes before returning to its initial configuration, however having advanced one step on the track. This sequence of conformational changes takes some random time $\tau$ to perform; the distribution of this stroke time is given by some probability density function $w(\tau)$. We control the motor by periodically modulating the fuel concentration. This leads to a periodically varying initiation rate $\gamma(t)$ while the stroke time distribution $w(\tau)$ is assumed to remain unaffected.

The motion of the Brownian stepper can be characterized by the instantaneous mean velocity $v(t)$ and the instantaneous effective diffusion constant $D_{\text{eff}}(t)$. Denoting by $N(t)$ the random number of steps performed up to time $t$, these quantities are defined as

$$v(t) = \mathcal{L}\frac{\mathrm{d}}{\mathrm{d}t}\langle N(t)\rangle \qquad \text{and} \qquad D_{\mathrm{eff}}(t) = \mathcal{L}^2\frac{\mathrm{d}}{\mathrm{d}t}\frac{\langle N(t)^2\rangle - \langle N(t)\rangle^2}{2} \tag{1}$$

where $\mathcal{L}$ is the length of one step of the Brownian stepper, which will be set to unity in the following examples. Due to the periodic modulation of the fuel concentration with period $\mathcal{T} = 2\pi/\Omega$, the velocity $v(t)$ and the effective diffusion coefficient $D_{\mathrm{eff}}(t)$ also become periodic functions for large $t$. We then may define corresponding quantities averaged over one period of the driving,

$$\bar{v} = \frac{1}{\mathcal{T}}\int_0^{\mathcal{T}} \mathrm{d}t\, v(t) \qquad \text{and} \qquad \bar{D}_{\mathrm{eff}} = \frac{1}{\mathcal{T}}\int_0^{\mathcal{T}} \mathrm{d}t\, D_{\mathrm{eff}}(t). \tag{2}$$

The averaged velocity and diffusion coefficient are used to determine the dimensionless Péclet number

$$Pe = \frac{\mathcal{L}\bar{v}}{\bar{D}_{\mathrm{eff}}}$$

which is best suited to characterize the regularity of the process. The higher the Péclet number the more regular is the motion independent of the step width or timescale. The Péclet number can be interpreted as the number of steps until the motion becomes randomized over one step length.

Let us briefly discuss the behaviour of the undriven model. Without driving the initiation of a step happens with a time independent rate $\gamma = \text{const}$. The stepping times then constitute a stationary renewal process, because the intervals between subsequent steps are independent of each other. These times between two subsequent steps are distributed according to

$$w_{\mathrm{tot}}(\tau) = (w_{\mathrm{init}} \circ w_{\mathrm{stroke}})(\tau) = \int_0^{\tau} \mathrm{d}\tau'\, w_{\mathrm{init}}(\tau')w_{\mathrm{stroke}}(\tau - \tau').$$

Therein $w_{\mathrm{init}}(\tau) = \gamma\exp(-\gamma\tau)$ is the distribution of initiation times while $w_{\mathrm{stroke}}(\tau) = w(\tau)$ is the distribution of stroke times. Introducing the mean and the variance of the step time

$$\langle\tau\rangle = \int_0^{\infty} \mathrm{d}\tau\, \tau w_{\mathrm{tot}}(\tau) \qquad \text{and} \qquad \operatorname{var}\tau = \int_0^{\infty} \mathrm{d}\tau\, \tau^2 w_{\mathrm{tot}}(\tau) - \langle\tau\rangle^2$$

the mean velocity and effective diffusion coefficient can be expressed as [13]

$$\bar{v} = \frac{\mathcal{L}}{\langle\tau\rangle} \qquad \text{and} \qquad \bar{D}_{\mathrm{eff}} = \frac{\mathcal{L}^2}{2}\frac{\operatorname{var}\tau}{\langle\tau\rangle^3}.$$

This leads to the Péclet number

$$Pe = 2\frac{\langle\tau\rangle^2}{\operatorname{var}\tau}. \tag{3}$$

In our further calculations we take the stroke time to be distributed according to a Γ-distribution.

$$w_{\mathrm{stroke}}(\tau) \equiv w_{n,T}(\tau) = \frac{1}{\Gamma(n)}\left(\frac{\tau n}{T}\right)^n \frac{\exp(-\frac{\tau n}{T})}{\tau}. \tag{4}$$

In figure 2 this waiting time distribution is illustrated for different values of $n$, which parametrizes different cases, ranging from an exponential distribution for $n = 1$ pertinent to a single rate process to a delta distribution for $n \to \infty$ corresponding to a fixed non-random stroke time. In particular, the mean stroke time is

$$\langle\tau_{\mathrm{stroke}}\rangle := \int_0^{\infty} \mathrm{d}\tau\, \tau w_{n,T}(\tau) = T$$

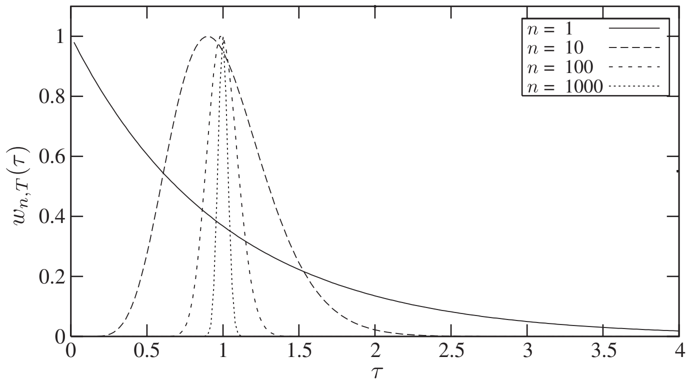


**Figure 2.** The gamma distribution equation (4) with mean $T = 1$ and two different values of $n$. The plots are non-normalized, but scaled such that the maximum value is unity.

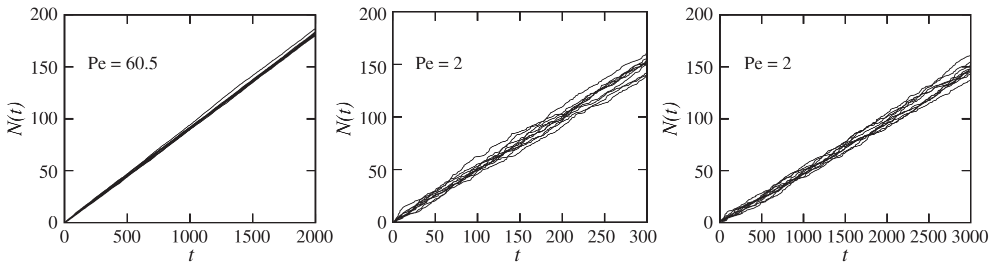


**Figure 3.** 20 different realizations of the number of steps as a function of time for different parameter sets. Left: highly ordered motion corresponding to large Péclet number ($\gamma = 1$, $T = 10$ and $n = 1009$). The middle and right figures show less coherent motion with small Péclet numbers for a fast (middle) and a slow stepper. (Middle: $\gamma = 1$, $T = 1$ and $n = 1$. Right: $\gamma = 0.1$, $T = 10$ and $n = 1$.)

while its variance reads

$$\mathrm{var}\,\tau_{\mathrm{stroke}} := \int_0^\infty \mathrm{d}\tau\, \tau^2 w_{n,T}(\tau) - \langle \tau_{\mathrm{stroke}} \rangle^2 = \frac{T^2}{n}.$$

Both can be varied independently by appropriately choosing $T$ and $n$. With this stroke time distribution the Péclet number equation (3) of the undriven Brownian stepper can be expressed as (see also [9])

$$Pe = 2\frac{(\frac{1}{\gamma} + T)^2}{\frac{1}{\gamma^2} + \frac{T^2}{n}}. \tag{5}$$

To illustrate the correspondence between Péclet number and regularity we have plotted in figure 3 20 different realizations of the number of steps as a function of time for three parameter sets, one with a high Péclet number and two with a low Péclet number.

In the following we will fix the value of $T = 5$ and use in examples $n = 100$ and 1000. Then in the undriven case $Pe$ from equation (5) ranges between 2 and 200 for $n = 100$ and 2000 for $n = 1000$ if $\gamma$ increases from zero to infinity. Later on, for the driven case we will find values which exceed these values by several orders of magnitude.

## 3. Theory

In [11, 12] a method was presented to evaluate the mean frequency and effective diffusion coefficient as defined by equations (1) in the asymptotic limit for periodically driven renewal processes. Such a periodic renewal process is defined by a time dependent waiting time distribution $w(\tau, t)$ which governs the time $\tau$ between two subsequent steps, if the previous step happened at time $t$. Due to the periodic driving the dependence on this time $t$ is also periodic with the period $\mathcal{T} = \frac{2\pi}{\Omega}$ of the periodic driving. In the case of the Brownian stepper model considered here, this time dependent waiting time distribution is given by the convolution of the time dependent activation time distribution and the stroke time distribution.

In what follows we briefly review the main ideas of this approach. Readers who are interested in the details of the calculation are referred to [12].

Let $p_k(t)$ be the probability to have made $k$ steps until time $t$ and $j_k(t)$ the probability density to make the $k$th step at time $t$. In the asymptotic limit these quantities obey the equations

$$p_k(t) = \int_0^\infty \mathrm{d}\tau\, j_{k-1}(t-\tau) z(\tau, t-\tau) \tag{6}$$

and

$$j_k(t) = \int_0^\infty \mathrm{d}\tau\, j_{k-1}(t-\tau) w(\tau, t-\tau) \tag{7}$$

where $z(\tau, t) = 1 - \int_0^\tau \mathrm{d}\tau'\, w(\tau', t)$ is the probability to wait longer than $\tau$ until the next step, provided the last step happened at $t$. This discrete microscopic dynamics can be embedded into a continuous envelope dynamics, governed by the probability density $\mathcal{P}(x, t)$, by assigning

$$p_k(t) = \frac{1}{\mathcal{L}} \int_{k-\frac{\mathcal{L}}{2}}^{k+\frac{\mathcal{L}}{2}} \mathrm{d}x\, \mathcal{P}(x, t). \qquad \text{and} \qquad j_k(t) = \mathcal{J}\left(k + \frac{\mathcal{L}}{2}, t\right), \tag{8}$$

where $\mathcal{J}$ is the probability current of the continuous embedding as defined by the continuity equation

$$\frac{\partial}{\partial t}\mathcal{P}(x, t) = -\frac{\partial}{\partial x}\mathcal{J}(x, t). \tag{9}$$

Assuming the envelope dynamics to be governed by the Kramers–Moyal equation

$$\frac{\partial}{\partial t}\mathcal{P}(x, t) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \kappa^{(n)}(t) \frac{\partial^n}{\partial x^n}\mathcal{P}(x, t) \tag{10}$$

one eventually deduces from the asymptotic equality of the instantaneous mean velocity and effective diffusion coefficient in both the discrete description and its continuous embedding the relation

$$\kappa^{(1)}(t) = v(t) \qquad \text{and} \qquad \kappa^{(2)}(t) = 2D_{\mathrm{eff}}(t).$$

Next the ansatz (8) is inserted into the microscopic dynamics, expressing the probability current $\mathcal{J}$ in terms of the probability distribution $\mathcal{P}$ according to equations (9) and (10) as

$$\mathcal{J}(x, t) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \kappa^{(n)}(t) \frac{\partial^{n-1}}{\partial x^{n-1}}\mathcal{P}(x, t). \tag{11}$$

The resulting terms $\mathcal{P}(k - \frac{\mathcal{L}}{2}, t - \tau)$ can be expressed in terms of $\frac{\partial^n}{\partial x^n}\mathcal{P}(x, t)|_{x=k}$ by performing a Taylor expansion and replacing the resulting time derivatives according to equation (10) by derivatives with respect to $x$. Equating the coefficients of $\mathcal{P}(k,t)$ and $\frac{\partial}{\partial x}\mathcal{P}(x,t)|_{x=k}$ one eventually arrives at

$$\int_0^\infty \mathrm{d}\tau\, v(t-\tau)z(\tau,t-\tau) = \mathcal{L} \tag{12}$$

$$\int_0^\infty \mathrm{d}\tau\, D_{\mathrm{eff}}(t-\tau)z(\tau,t-\tau) = 2\int_0^\infty \mathrm{d}\tau\, v(t-\tau)\int_0^\tau \mathrm{d}\tau'\, v(t-\tau')z(\tau,t-\tau) - \mathcal{L}^2. \tag{13}$$

In [11, 12] also the special case of a periodically driven renewal process was considered, where the time intervals between subsequent events are governed by a rate process with a periodically time dependent rate $\gamma(t)$ followed by a waiting time, which is distributed according to some waiting time density $w(\tau)$. This is exactly the situation we face in our model for the Brownian stepper. Denoting by $z(\tau) = 1 - \int_0^\tau \mathrm{d}\tau'\, w(\tau)$ the probability that a stroke will take longer than $\tau$, equations (12) and (13) can be further simplified. This eventually leads to [12]

$$v(t) + \gamma(t)\int_0^\infty \mathrm{d}\tau\, v(t-\tau)z(\tau) = \mathcal{L}\gamma(t) \tag{14}$$

and

$$\begin{aligned} D_{\mathrm{eff}}(t) + \gamma(t)\int_0^\infty \mathrm{d}\tau\, D_{\mathrm{eff}}(t-\tau)z(\tau) \\ = -\frac{\mathcal{L}^2}{2}\gamma(t) + \mathcal{L}v(t) + \gamma(t)\int_0^\infty \mathrm{d}\tau\, v(t-\tau)\int_0^\tau \mathrm{d}\tau'\, v(t-\tau')z(\tau). \end{aligned} \tag{15}$$

The periodic solutions of the inhomogenous linear integral equations (14) and (15) can be numerically obtained in Fourier space using a linear solver like LAPACK. From the periodic instantaneous mean velocity $v(t)$ and instantaneous effective diffusion coefficient $D_{\mathrm{eff}}(t)$, the desired mean velocity $\bar{v}$ and effective diffusion coefficient $\bar{D}_{\mathrm{eff}}$ are obtained by performing a period average according to equations (2).

## 4. Results

To present specific results we consider two types of periodic driving, namely a dichotomic activation rate

$$\gamma_{\mathrm{d}}(t) = \begin{cases} r_1 & \text{if } t \in [n\mathcal{T}, (n+\frac{1}{2})\mathcal{T}) \\ r_2 & \text{if } t \in [(n+\frac{1}{2})\mathcal{T}, (n+1)\mathcal{T}) \end{cases} \tag{16}$$

where $\mathcal{T}$ is the period of the modulation and a harmonic driving

$$\gamma_{\mathrm{h}}(t) = \frac{r_1 + r_2}{2} + \frac{r_1 - r_2}{2}\cos\Omega t. \tag{17}$$

First let us consider a situation where the periodic driving (16) induces a change in the attachment rates $\gamma$ by one order of magnitude. i.e. $r_2/r_1 = 10$. Figure 4 shows results of the theory presented in the previous section according to equations (14) and (15) and compares them with mean velocities $\bar{v}$ and diffusion coefficients $\bar{D}_{\mathrm{eff}}$ obtained from simulations of an ensemble of 100 000 Brownian steppers. Both curves agree within simulation precision. Deviations occur due to finite simulation times.

The mean velocity exhibits a 1:1 synchronization with the periodic driving near $\Omega \approx 1$. In the region of synchronization the diffusion coefficient becomes minimal. The steppers follow the periodic driving with high precision.

A second synchronization window appears for smaller frequencies, i.e. for longer periods. During half of the period when the excitation rate $\gamma(t)$ is large the stepper succeeds in

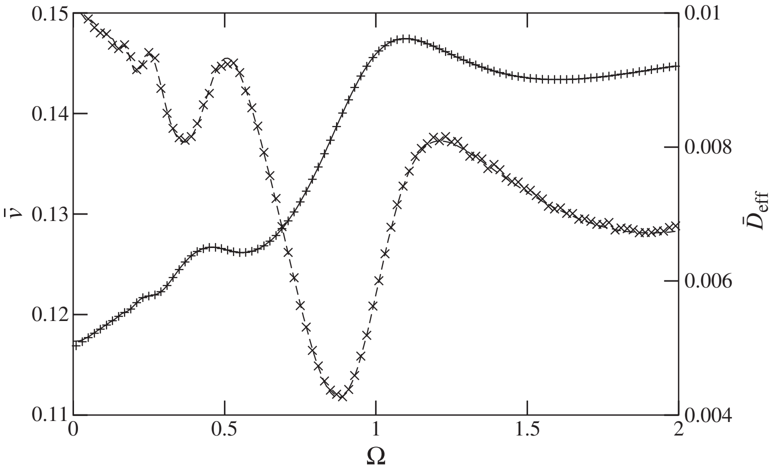


**Figure 4.** Mean frequency $\bar{v}$ (solid line and +) and effective diffusion coefficient $\bar{D}_{\text{eff}}$ (dashed line and $\times$) of the dichotomically driven model with $r_1 = 0.1$, $r_2 = 1.0$, $T = 5.0$ and $n = 20$. The step length $\mathcal{L}$ is chosen to be 1. Symbols are simulations of the driven renewal process, while the lines correspond to the theory equations (14) and (15).

performing two steps. Again the diffusion coefficients become smaller compared to regions without synchronization.

Figure 4 also allows the discussion of the limits of fast driving $\Omega \to \infty$ and slow driving $\Omega \to 0$. In the case of fast driving the averaged velocity and diffusion coefficient of the steppers coincide with the values of the undriven case if $\gamma(t)$ is replaced by the arithmetic mean of the rate. i.e. $\gamma(t) \to (r_1 + r_2)/2$. Conversely, if the switching frequency is vanishingly small one can average between the two velocities and diffusion coefficients of the undriven situations. Therefore, the mean velocity becomes $v = (v_1 + v_2)/2$, where $v_1$ is its stationary value with rate $r_1$, respectively $v_2$ and $r_2$. The limiting values of the effective diffusion coefficient can be obtained in the same way.

In order to amplify synchronization we increase the difference of the two fuel attachment rates and decrease the variance of the stroke time. We now put the ratio $r_2/r_1 = 100$. Figure 5 shows again the dependence of the mean velocity, effective diffusion coefficient and Péclet number on the driving frequency in the case of dichotomic driving (16) for these rates.

We observe different regions of frequency synchronization. These regions show a rational relation between driving frequency and step frequency which is proportional to the velocity. The frequency locking is accompanied by a low effective diffusion. This leads to a high Péclet number, thus in these regions the motion of the molecular motor is very regular. Between these regions the effective diffusion coefficient strongly increases, showing a less coherent motion of the motor.

The behaviour of the harmonically driven motor (17) is qualitatively the same as the behaviour of the dichotomic (16) motor. Again, we obtain very high Péclet numbers with locked velocity and low diffusion if the motion of the stepper is synchronized with the periodic drive. However, as seen in (figure 6), the harmonic driving allows a more precise tuning of the driven motor, since the stepper also exhibits a 3:2 synchronization regime, i.e. three steps of the motor lie within two periods of the driving. Such behaviour was not observed for the dichotomically driven system.

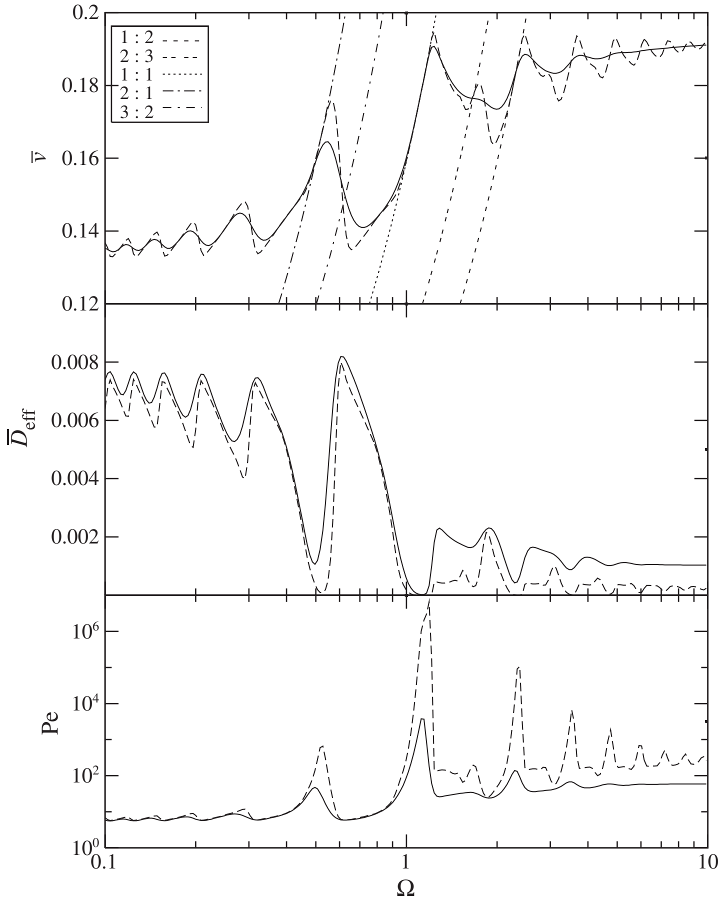


**Figure 5.** Mean velocity (top), effective diffusion coefficient (middle) and Péclet number (bottom) as a function of driving frequency of the dichotomically driven system. Two situations with different dispersions of the stroke times are shown. Parameters: $r_1 = 0.1, r_2 = 10.0, T = 5.0, n = 100$ (solid line); $r_1 = 0.1, r_2 = 10.0, T = 5.0, n = 1000$ (dashed line). The corresponding two values for the undriven system with initiation rate $\gamma = \frac{r_1+r_2}{2}$ are $\bar{v} \approx 0.192$ (0.192), $\bar{D}_{\mathrm{eff}} \approx 0.001$ (0.000 23) and $Pe = 93$ (416) (values in parentheses: second parameter set). The additional curves in the top figure indicate perfect synchronization between the motion of the stepper and the periodic drive; the corresponding ratios are shown in the key.

In the high frequency limit both driving types lead to the same behaviour; however, for low frequencies the velocity in the harmonically driven system is higher, while the effective diffusion coefficient is much lower, leading to an increased Péclet number compared to the dichotomically driven system.

Next let us compare the periodically driven system in the presence and in the absence of synchronization with the corresponding undriven system (see figures 7). To this end we have chosen the excitation rate of the undriven system between the maximum rate $r_2$ and the minimum rate $r_1$ of the driven system, such that the motion is most regular, i.e. the Péclet number is maximal. This optimal value $r_{\mathrm{opt}}$ is depicted in both figures (dashed lines). The Péclet numbers for the driven system and the undriven system are shown as a function of the variance of the stroke time for two different driving frequencies, one lying within the 1:1

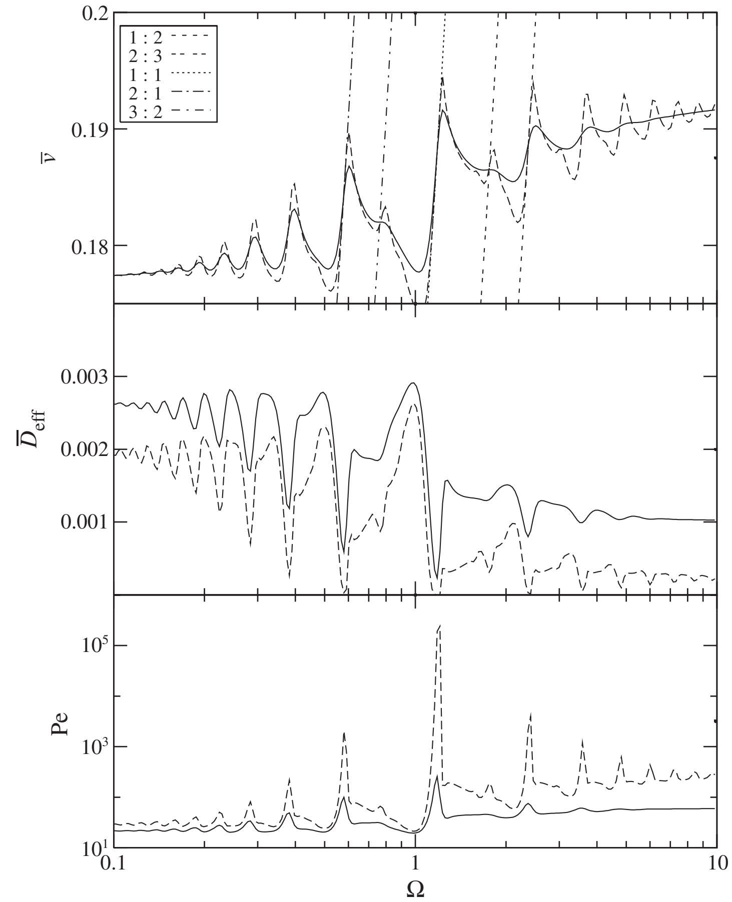


**Figure 6.** Mean velocity (top), effective diffusion coefficient (middle) and Péclet number (bottom) as a function of driving frequency for the harmonically driven system. Two situations with different dispersions of the stroke times. Parameters: $r_1 = 0.1$, $r_2 = 10.0$, $T = 5.0$, $n = 100$ (solid line); $r_1 = 0.1$, $r_2 = 10.0$, $T = 5.0$, $n = 1000$ (dashed line). The additional curves in the top figure indicate perfect synchronization between the motion of the stepper and the periodic drive, the corresponding ratios are shown in the key.

synchronization regime and the other in a region without synchronization. We see that in the case of synchronization the coherence of motion can be significantly increased. Out of synchrony the periodic drive reduces the level of the regularity of the motion. For smaller values of the stroke time variance the optimal rate is the maximal rate $r_{\mathrm{opt}} = r_2$. In the undriven case a quick attachment of fuel molecules and small variances of stroke times result in a nearly periodic motion. If this situation is perturbed periodically by changing to a smaller rate $r_1 \ll r_{\mathrm{opt}}$ much disorder is added to the motion, since while remaining in the state with small rate the dispersion is $\propto 1/r_1$. This leads to the significant decrease of the Péclet number out of synchrony.

The periodic drive might be an instrument for probing the characteristic times of the configurational change. In case of a significant periodic variation of fuel attachment rate $\gamma(t)$ the synchronization between the motor and the periodic driving, as indicated by a high Péclet number, is observed for driving frequencies that are equal to or slightly less than integer

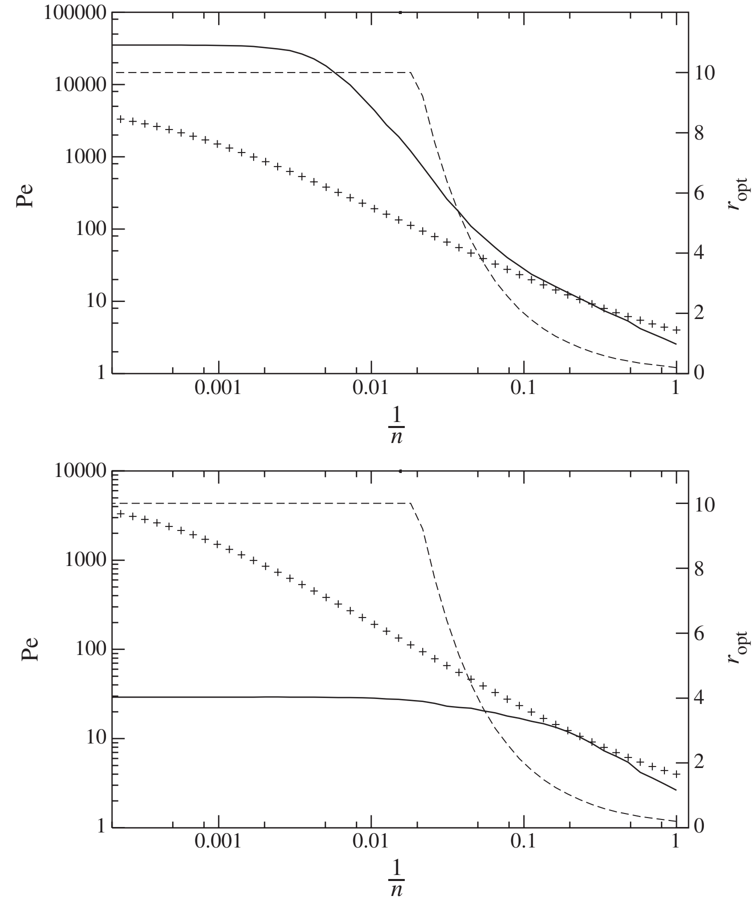


**Figure 7.** Comparison between the Péclet numbers of the dichotomically driven (solid line) and the undriven system (+) as a function of the relative variance of the stroke time $\frac{\mathrm{var}\,\tau_{\mathrm{stroke}}}{\langle\tau_{\mathrm{stroke}}\rangle^2} = \frac{1}{n}$. Top: 1:1 synchronization regime. Parameters: $r_1 = 0.1, r_2 = 10.0, T = 5.0, \omega = 1.1$. Bottom: out of synchronization. Parameters: $r_1 = 0.1, r_2 = 10.0, T = 5.0, \omega = 0.8$. The rate for the undriven system is chosen between $r_1$ and $r_2$ such that the Péclet number is maximized. This optimal rate $r_{\mathrm{opt}}$ is indicated by the dashed line.

multiples of $2\pi$ times the mean stroke time $T$ of the motor. Thus by tuning the driving frequency and measuring the Péclet number one can deduce the mean stroke time $T$. This is presented in figure 8 where the bright regions indicate high Péclet numbers as a function of the frequency of the periodic drive $\Omega$ and the mean stroke time $T$. One sees immediately the several regions of $n$:$m$ synchronization which might be used to determine the mean stroke time. Both types of driving, i.e. dichotomic (left) and harmonic (right), exhibit qualitatively the same behaviour. Again, the harmonic drive allows a finer tuning of the stepper.

## 5. Conclusion

We have shown that the periodic driving, being dichotomic or harmonic, may regularize the motion of the molecular motor. For this purpose we have studied a unidirectionally moving Brownian stepper. Its single step consists of an exponentially distributed waiting

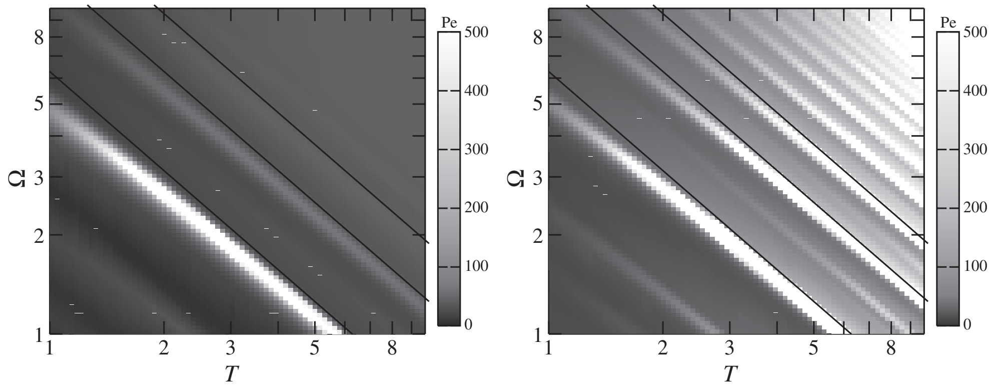


**Figure 8.** Péclet number as a function of driving frequency $\Omega$ and mean stroke time $T$ for the dichotomically (left) and harmonically driven (right) model with $r_1 = 0.1$, $r_2 = 10$ and $n = 100$. The solid line correspond to $\Omega = 2\pi/T$, $\Omega = 4\pi/T$ and $\Omega = 6\pi/T$. The synchronization regions as indicated by highest Péclet number may thus be used to determine the mean stroke time $T$.

time for the attachment of fuel molecules and a second gamma-distributed time modelling the configurational change performing the forward motion. We have assumed that the attachment rate of the fuel molecules is periodically varied by a periodic modulation of the fuel concentration.

The coherence of the motion is measured by the Péclet number. It gives the number of steps until fluctuations smear out the position of the stepper over one step length. Maximal Péclet numbers were found if the motor synchronizes to the periodic drive. Several regions of $n{:}m$ synchronizations with locked velocity and small effective diffusion were found. Conversely, out of synchrony the stepper performs more disordered motion compared with the stationary undriven case where the attachment rate is optimized to yield maximal Péclet numbers. We have further shown that periodic driving can be used to work out parameters of the Brownian stepper like the mean stroke time.

We believe that more complex models than the Brownian stepper can be synchronized to high Péclet numbers as well. Therefore, the proposed periodic driving of molecular motors might be a new technique for improvement and regularization of the random motion.

## Acknowledgment

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## References

[1] Magnasco M O 1993 *Phys. Rev. Lett.* **71** 1477
[2] Reimann P 2002 *Phys. Rep.* **361** 57
[3] Parrondo J M R and de Cisneros B J 2002 *Appl. Phys.* A **75** 179
[4] Anishchenko V, Neiman A, Astakhov A, Vadiavasova T and Schimansky-Geier L 2002 *Chaotic and Stochastic Processes in Dynamic Systems* (*Springer-Series on Synergetics*) (Berlin: Springer)
[5] Fischer S, Windshügel B, Horak D, Holmes K C and Smith J C 2005 *Proc. Natl Acad. Sci. USA* **102** 6873
[6] Ökten Z, Churchman L S, Rock R S and Spudich J A 2004 *Nat. Struct. Mol. Biol.* **11** 884
[7] Lister I, Schmitz S, Walker M, Trinick J, Buss F, Weigel C and Kendrick-Jones J 2004 *EMBO J.* **23** 1729

[8] Astumian R D and Hänggi P 2002 *Phys. Today* **55** (11) 33
[9] Bier M 2003 *Phys. Rev. Lett.* **91** 148104
[10] Freund J A and Schimansky-Geier L 1999 *Phys. Rev.* E **60** 1304
[11] Prager T and Schimansky-Geier L 2005 *Phys. Rev.* E **71** 031112
[12] Prager T and Schimansky-Geier L 2005 *J. Stat. Phys.* submitted (*Preprint* cond-mat/0509485)
[13] Cox D R 1962 *Renewal Theory* (London: Methuen)